PHIẾU BÀI TẬP CĂN THỨC BẬC HAI SỐ 4

Bài 1: Giải các phương trình sau

a) $\sqrt{x^2 - 4x + 4} = 5$

b) $3\sqrt{4x-8} - \frac{2}{5}\sqrt{25x-50} + \sqrt{9x-18} = 7$

c) $\sqrt{1-3x} = 4$

d) $\sqrt{4-5x} = 2-5x$

e) $\sqrt{4-3x} = \sqrt{2x-1}$

f) $\sqrt{x^2-25} - 6 = 2.\sqrt{x-5} - 3.\sqrt{x+5}$

g) $\sqrt{x^2-16} - 3\sqrt{x+4} = 0$

h) $\sqrt{x+3+4\sqrt{x-1}} = 3$

i) $\sqrt{4(x^2-2x+1)} = 6$

j) $\sqrt{x^2-4x+4} - 2x + 5 = 0$

k) $\frac{x+2\sqrt{x}}{\sqrt{x}-1} = 8$

l) $\sqrt{x-1} + \sqrt{x-2} = \sqrt{3x-1}$

Giải:

a) $\sqrt{x^2 - 4x + 4} = 5$ $\qquad TXD: x \in R$

$\Leftrightarrow \sqrt{(x-2)^2} = 5$

$\Leftrightarrow |x-2| = 5$

$\Leftrightarrow \begin{bmatrix} x-2 = 5 \\ x-2 = -5 \end{bmatrix} \Leftrightarrow \begin{bmatrix} x = 7 \ (tm) \\ x = -3 \ (tm) \end{bmatrix}$

b) $3\sqrt{4x-8} - \frac{2}{5}\sqrt{25x-50} + \sqrt{9x-18} = 7$

$(DK: x \geq 2)$

$3\sqrt{4(x-2)} - \frac{2}{5}\sqrt{25(x-2)} + \sqrt{9(x-2)} = 7$

$\Leftrightarrow 6\sqrt{x-2} - 2\sqrt{x-2} + 3\sqrt{x-2} = 7$

$\Leftrightarrow 7\sqrt{x-2} = 7$

$\Leftrightarrow \sqrt{x-2} = 1$

$\Leftrightarrow x-2 = 1$

$\Leftrightarrow x = 3 \ (tm)$

c) $\sqrt{1-3x} = 4$ $\qquad (DK: x \leq \frac{1}{3})$

$\Leftrightarrow 1-3x = 16$

$\Leftrightarrow -3x = 15$

$\Leftrightarrow x = -5 \ (tm)$

e) $\sqrt{4-3x} = \sqrt{2x-1}$ $\quad (DK: \frac{1}{2} \leq x \leq \frac{4}{3})$

$\Leftrightarrow \sqrt{4-3x}^2 = \sqrt{2x-1}^2$

$\Leftrightarrow 4-3x = 2x-1$

$\Leftrightarrow -5x = -5$

$\Leftrightarrow x = 1 \ (tm)$

f) $\sqrt{x^2-25} - 6 = 2.\sqrt{x-5} - 3.\sqrt{x+5}$

$(DK: x \geq 5)$

d) $\sqrt{4-5x} = 2-5x$ $\qquad (DK: x \leq \frac{4}{5})$

$\Leftrightarrow \begin{cases} 2-5x \geq 0 \\ \sqrt{4-5x}^2 = (2-5x)^2 \end{cases}$

$\Leftrightarrow \begin{cases} x \leq \frac{2}{5} \\ 4-5x = (2-5x)^2 \end{cases}$

$\Leftrightarrow \begin{cases} x \leq \frac{2}{5} \\ 4-5x = (2-5x)^2 \end{cases}$

$\Leftrightarrow \sqrt{(x-5)(x+5)} - 6 - 2.\sqrt{x-5} + 3.\sqrt{x+5} = 0$

$\Leftrightarrow (\sqrt{(x-5)(x+5)} - 2.\sqrt{x-5}) + (3.\sqrt{x+5} - 6) = 0$

$\Leftrightarrow \sqrt{x-5}(\sqrt{x+5} - 2) + 3(\sqrt{x+5} - 2) = 0$

$\Leftrightarrow (\sqrt{x-5} + 3)(\sqrt{x+5} - 2) = 0 \qquad (*)$

$Vi\sqrt{x-5} \ge 0 \qquad \forall x \in TXD$

$\Rightarrow \sqrt{x-5} + 3 \ge 3 > 0 \quad \forall x \in TXD$

$(*) \Leftrightarrow \sqrt{x+5} - 2 = 0$

$\Leftrightarrow x + 5 = 4$

$\Leftrightarrow x = -1 \quad (l)$

$\Leftrightarrow \begin{cases} x \le \dfrac{2}{5} \\ 4 - 5x = 4 - 20x + 25x^2 \end{cases}$

$\Leftrightarrow \begin{cases} x \le \dfrac{2}{5} \\ 25x^2 - 15x = 0 \end{cases} \Leftrightarrow \begin{cases} x \le \dfrac{2}{5} \\ 5x(5x - 3) = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x \le \dfrac{2}{5} \\ \left[\begin{array}{l} x = 0 \\ x = \dfrac{3}{5} \end{array}\right. \end{cases} \Leftrightarrow x = 0 \ (tm)$

h) $\sqrt{x+3+4\sqrt{x-1}} = 3$

$(DK : x \ge 1)$

$\Leftrightarrow \sqrt{x+3+4\sqrt{x-1}}^2 = 3^2$

$\Leftrightarrow x + 3 + 4\sqrt{x-1} = 9$

$\Leftrightarrow x - 6 = 4\sqrt{x-1}$

$\Leftrightarrow \begin{cases} x - 6 \ge 0 \\ x - 6 = 4\sqrt{x-1} \end{cases} \Leftrightarrow \begin{cases} x \ge 6 \\ (x-6)^2 = (4\sqrt{x-1})^2 \end{cases}$

$\Leftrightarrow \begin{cases} x \ge 6 \\ x^2 - 12x + 36 = 16.(x-1) \end{cases}$

$\Leftrightarrow \begin{cases} x \ge 6 \\ x^2 - 28x + 52 = 0 \end{cases} \Leftrightarrow \begin{cases} x \ge 6 \\ x^2 - 2x - 26x + 52 = 0 \end{cases}$

g) $\sqrt{x^2 - 16} - 3\sqrt{x+4} = 0$

$(DK : x = -4; x \ge 4)$

$\Leftrightarrow \sqrt{x^2 - 4^2} - 3\sqrt{x+4} = 0$

$\Leftrightarrow \sqrt{(x-4)(x+4)} - 3\sqrt{x+4} = 0$

$\Leftrightarrow \sqrt{x+4}.(\sqrt{x-4} - 3) = 0$

$\Leftrightarrow \sqrt{x+4}.(\sqrt{x-4} - 3) = 0$

$\Leftrightarrow \left[\begin{array}{l} \sqrt{x+4} = 0 \\ \sqrt{x-4} - 3 = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x + 4 = 0 \\ x - 4 = 9 \end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l} x = -4 \quad (tm) \\ x = 13 \quad (tm) \end{array}\right.$

$\Leftrightarrow \begin{cases} x \ge 6 \\ x(x-2) - 26(x-2) = 0 \end{cases} \Leftrightarrow \begin{cases} x \ge 6 \\ (x-26).(x-2) = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x \ge 6 \\ \left[\begin{array}{l} x - 26 = 0 \\ x - 2 = 0 \end{array}\right. \end{cases} \Leftrightarrow \begin{cases} x \ge 6 \\ \left[\begin{array}{l} x = 26 \\ x = 2 \end{array}\right. \end{cases} \Leftrightarrow x = 26 \ (tm)$

i) $\sqrt{4(x^2 - 2x + 1)} = 6$

$(TXD : x \in R)$

$\Leftrightarrow \sqrt{4(x-1)^2} = 6$

$\Leftrightarrow \sqrt{[2(x-1)]^2} = 6$

$\Leftrightarrow |2(x-1)| = 6$

$\Leftrightarrow \left[\begin{array}{l} 2(x-1) = 6 \\ 2(x-1) = -6 \end{array}\right. \qquad \Leftrightarrow \left[\begin{array}{l} x - 1 = 3 \\ x - 1 = -3 \end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l} x = 4 \quad (tm) \\ x = -2 \quad (tm) \end{array}\right.$

j) $\sqrt{x^2 - 4x + 4} - 2x + 5 = 0$

k) $\dfrac{x + 2\sqrt{x}}{\sqrt{x} - 1} = 8$

$(TXD : x \in R)$

$\Leftrightarrow \sqrt{x^2 - 4x + 4} = 2x - 5$

$\Leftrightarrow \sqrt{(x-2)^2} = 2x - 5$

$\Leftrightarrow |x - 2| = 2x - 5$

$\Leftrightarrow \left[\begin{array}{l} x - 2 = 2x - 5 \quad khi \ x \ge 2 \\ x - 2 = -(2x - 5) \quad khi \ x < 2 \end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l} x = 3 \quad (tm) \\ x = \dfrac{7}{3} \quad (l) \end{array}\right. \Leftrightarrow x = 3 \quad (tm)$

$(DK : x \ge 0 ; x \ne 1)$

$\Leftrightarrow x + 2\sqrt{x} = 8.(\sqrt{x} - 1)$

$\Leftrightarrow x + 2\sqrt{x} = 8\sqrt{x} - 8$

$\Leftrightarrow x - 6\sqrt{x} + 8 = 0$

$\Leftrightarrow x - 2.\sqrt{x}.3 + 9 - 1 = 0$

$\Leftrightarrow (\sqrt{x} - 3)^2 - 1 = 0$

$\Leftrightarrow (\sqrt{x} - 3 - 1)(\sqrt{x} - 3 + 1) = 0$

$\Leftrightarrow (\sqrt{x} - 4)(\sqrt{x} - 2) = 0$

$\Leftrightarrow \left[\begin{array}{l} \sqrt{x} - 4 = 0 \\ \sqrt{x} - 2 = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \sqrt{x} = 4 \\ \sqrt{x} = 2 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = 16 \quad (tm) \\ x = 4 \quad (tm) \end{array}\right.$

l) $\sqrt{x-1} + \sqrt{x-2} = \sqrt{3x-1} \qquad (DK : x \ge 2)$

$\Leftrightarrow (\sqrt{x-1} + \sqrt{x-2})^2 = \sqrt{3x-1}^2$

$\Leftrightarrow x - 1 + 2\sqrt{x-1}.\sqrt{x-2} + x - 2 = 3x - 1$

$\Leftrightarrow 2\sqrt{x-1}.\sqrt{x-2} + 2x - 3 = 3x - 1$

$\Leftrightarrow 2\sqrt{x-1}.\sqrt{x-2} = x + 2$

$\Leftrightarrow (2\sqrt{x-1}.\sqrt{x-2})^2 = (x+2)^2$

$\Leftrightarrow 4(x-1)(x-2) = (x+2)^2$

$\Leftrightarrow 4(x^2 - 3x + 2) = (x^2 + 4x + 4)$

$\Leftrightarrow 3x^2 - 16x + 4 = 0$

$\Leftrightarrow x^2 - \dfrac{16}{3}x + \dfrac{4}{3} = 0$

$\Leftrightarrow x^2 - 2.\dfrac{8}{3}x + \dfrac{64}{9} - \dfrac{52}{9} = 0$

$\Leftrightarrow (x - \dfrac{8}{3})^2 - \left(\dfrac{\sqrt{52}}{3}\right)^2 = 0$

$\Leftrightarrow \left[\begin{array}{l} x - \dfrac{8}{3} = \dfrac{\sqrt{52}}{3} \\ x - \dfrac{8}{3} = -\dfrac{\sqrt{52}}{3} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\sqrt{52} + 8}{3} \quad (tm) \\ x = \dfrac{8 - \sqrt{52}}{3} \quad (l) \end{array}\right.$

Bài 2: Cho biểu thức:

$$Q = \frac{3x - 3\sqrt{x} - 3}{x + \sqrt{x} - 2} - \frac{\sqrt{x} + 1}{\sqrt{x} + 2} + \frac{\sqrt{x} - 2}{1 - \sqrt{x}}$$

a) Rút gọn biểu thức

$$Q = \frac{3x - 3\sqrt{x} - 3}{x + \sqrt{x} - 2} - \frac{\sqrt{x} + 1}{\sqrt{x} + 2} + \frac{\sqrt{x} - 2}{1 - \sqrt{x}}$$

$(DK: \quad x \geq 0, \quad x \neq 1)$

$$= \frac{3x - 3\sqrt{x} - 3}{x + 2\sqrt{x} - \sqrt{x} - 2} - \frac{\sqrt{x} + 1}{\sqrt{x} + 2} + \frac{\sqrt{x} - 2}{1 - \sqrt{x}}$$

$$= \frac{3x - 3\sqrt{x} - 3}{\sqrt{x}(\sqrt{x} + 2) - (\sqrt{x} + 2)} - \frac{\sqrt{x} + 1}{\sqrt{x} + 2} + \frac{\sqrt{x} - 2}{1 - \sqrt{x}}$$

$$= \frac{3x - 3\sqrt{x} - 3}{(\sqrt{x} - 1)(\sqrt{x} + 2)} - \frac{\sqrt{x} + 1}{\sqrt{x} + 2} + \frac{\sqrt{x} - 2}{1 - \sqrt{x}}$$

$$= \frac{(3x - 3\sqrt{x} - 3) - (\sqrt{x} + 1)(\sqrt{x} - 1) - (\sqrt{x} + 2)(\sqrt{x} - 2)}{(\sqrt{x} - 1)(\sqrt{x} + 2)}$$

$$= \frac{(3x - 3\sqrt{x} - 3) - (x - 1) - (x - 4)}{(\sqrt{x} - 1)(\sqrt{x} + 2)}$$

$$= \frac{x - 3\sqrt{x} + 2}{(\sqrt{x} - 1)(\sqrt{x} + 2)} = \frac{x - \sqrt{x} - 2\sqrt{x} + 2}{(\sqrt{x} - 1)(\sqrt{x} + 2)}$$

$$= \frac{\sqrt{x}(\sqrt{x} - 1) - 2(\sqrt{x} - 1)}{(\sqrt{x} - 1)(\sqrt{x} + 2)}$$

$$= \frac{(\sqrt{x} - 2)(\sqrt{x} - 1)}{(\sqrt{x} - 1)(\sqrt{x} + 2)} = \frac{\sqrt{x} - 2}{\sqrt{x} + 2}$$

b) Tính giá trị của Q khi x $= 4 + 2\sqrt{3}$

$x = 4 + 2\sqrt{3} = 3 + 2.\sqrt{3}.1 + 1$

$= \sqrt{3}^2 + 2.\sqrt{3}.1 + 1^2$

$= (\sqrt{3} + 1)^2$

$\Rightarrow \sqrt{x} = \sqrt{(\sqrt{3} + 1)^2} = \left|\sqrt{3} + 1\right| = \sqrt{3} + 1$

$Q = \dfrac{\sqrt{x} - 2}{\sqrt{x} + 2}$

$= \dfrac{\sqrt{3} + 1 - 2}{\sqrt{3} + 1 + 2} = \dfrac{\sqrt{3} - 1}{\sqrt{3} + 3}$

c) Tìm x để Q = 1/3

$Q = \dfrac{\sqrt{x} - 2}{\sqrt{x} + 2} = \dfrac{1}{3}$

$\Leftrightarrow 3.(\sqrt{x} - 2) = \sqrt{x} + 2$

$\Leftrightarrow 3\sqrt{x} - 6 = \sqrt{x} + 2$

$\Leftrightarrow 3\sqrt{x} - 6 - \sqrt{x} - 2 = 0$

$\Leftrightarrow 2\sqrt{x} - 8 = 0$

$\Leftrightarrow \sqrt{x} = 4$

$\Leftrightarrow x = 16 \quad (tm)$

d) Tìm x để cho Q > ½

$Q = \dfrac{\sqrt{x} - 2}{\sqrt{x} + 2} > \dfrac{1}{2}$

$\Leftrightarrow \dfrac{2.(\sqrt{x} - 2) - (\sqrt{x} + 2)}{2(\sqrt{x} + 2)} > 0$

$\Leftrightarrow \dfrac{\sqrt{x} - 6}{2(\sqrt{x} + 2)} > 0 \qquad (*)$

$Vi \ \sqrt{x} \geq 0 \qquad \forall x \in TXD$

$\Rightarrow \sqrt{x} + 2 \geq 2 > 0 \qquad \forall x \in TXD$

$\Rightarrow (*) \Leftrightarrow \sqrt{x} - 6 > 0$

$\Leftrightarrow \sqrt{x} > 6$

$\Leftrightarrow x > 36$

Kết hợp với điều kiện xác định, ta được: $x > 36$

e) Tìm x để |Q| = Q

Để |Q| = Q thì $Q \geq 0$

$\Leftrightarrow \dfrac{\sqrt{x}-2}{\sqrt{x}+2} \geq 0$ (**)

$Vi \sqrt{x} \geq 0 \qquad \forall x \in TXD$

$\Rightarrow \sqrt{x}+2 \geq 2 > 0 \qquad \forall x \in TXD$

$\Rightarrow (**) \Leftrightarrow \sqrt{x}-2 \geq 0$

$\Leftrightarrow x \geq 4$

Kết hợp với điều kiện xác định, ta được: $x \geq 4$

f) Tìm x để |Q| = - Q

Để |Q| = - Q thì $Q < 0$

$\Leftrightarrow \dfrac{\sqrt{x}-2}{\sqrt{x}+2} < 0$ (***)

$Vi \sqrt{x} \geq 0 \qquad \forall x \in TXD$

$\Rightarrow \sqrt{x}+2 \geq 2 > 0 \qquad \forall x \in TXD$

$\Rightarrow (***) \Leftrightarrow \sqrt{x}-2 < 0$

$\Leftrightarrow x < 4$

Kết hợp với điều kiện xác định, ta được: $x \geq 0; \ x \neq 1; \ x < 4$

g) Tìm x để |Q| > Q

Để |Q| > Q thì $Q < 0$

$\Leftrightarrow \dfrac{\sqrt{x}-2}{\sqrt{x}+2} < 0$ (***)

$Vi \sqrt{x} \geq 0 \qquad \forall x \in TXD$

$\Rightarrow \sqrt{x}+2 \geq 2 > 0 \qquad \forall x \in TXD$

$\Rightarrow (***) \Leftrightarrow \sqrt{x}-2 < 0$

$\Leftrightarrow x < 4$

Kết hợp với điều kiện xác định, ta được: $x \geq 0; \ x \neq 1; \ x < 4$

h) Tìm x để |Q| > - Q

Để |Q| > - Q thì $Q > 0$

$\Leftrightarrow \dfrac{\sqrt{x}-2}{\sqrt{x}+2} > 0$ (**)

$Vi \sqrt{x} \geq 0 \qquad \forall x \in TXD$

$\Rightarrow \sqrt{x}+2 \geq 2 > 0 \qquad \forall x \in TXD$

$\Rightarrow (**) \Leftrightarrow \sqrt{x}-2 > 0$

$\Leftrightarrow x > 4$

Kết hợp với điều kiện xác định, ta được: $x > 4$

i) Tìm $x \in Z$ để $Q \in Z$

$Q \in Z \Rightarrow \dfrac{\sqrt{x}-2}{\sqrt{x}+2} \in Z$

$hay \ Q = \dfrac{\sqrt{x}+2-4}{\sqrt{x}+2} \in Z$

$\Rightarrow 1 - \dfrac{4}{\sqrt{x}+2} \in Z$

$\Rightarrow \dfrac{4}{\sqrt{x}+2} \in Z \Rightarrow 4 \vdots \sqrt{x}+2$

$\Rightarrow \sqrt{x}+2 \in U(4)$

$Vi \sqrt{x} \geq 0 \qquad \forall x \in TXD$

$\Rightarrow \sqrt{x}+2 \geq 2 > 0 \qquad \forall x \in TXD$

$\Rightarrow \sqrt{x}+2 \in \{2;4\}$

$TH1: \sqrt{x}+2 = 2 \Leftrightarrow \sqrt{x} = 0 \Leftrightarrow x = 0 \ (tm)$

$TH2: \sqrt{x}+2 = 4 \Leftrightarrow \sqrt{x} = 2 \Leftrightarrow x = 4 \ (tm)$

j) Tìm x để $Q \in Z$

$Q \in Z \Rightarrow \dfrac{\sqrt{x}-2}{\sqrt{x}+2} \in Z$

$hay \ Q = \dfrac{\sqrt{x}+2-4}{\sqrt{x}+2} \in Z$

$\Rightarrow 1 - \dfrac{4}{\sqrt{x}+2} \in Z$

$\Rightarrow \dfrac{4}{\sqrt{x}+2} \in Z \Rightarrow 4 \vdots \sqrt{x}+2$

$\Rightarrow \sqrt{x}+2 \in U(4)$

$Vi \sqrt{x} \geq 0 \qquad \forall x \in TXD$

$\Rightarrow \sqrt{x}+2 \geq 2 > 0 \qquad \forall x \in TXD$

$\Rightarrow \sqrt{x}+2 \in \{2;4\}$

$TH1: \sqrt{x}+2 = 2 \Leftrightarrow \sqrt{x} = 0 \Leftrightarrow x = 0 \ (tm)$

$TH2: \sqrt{x}+2 = 4 \Leftrightarrow \sqrt{x} = 2 \Leftrightarrow x = 4 \ (tm)$

k) Tìm x để phương trình Q = m có nghiệm.

$$\frac{\sqrt{x}-2}{\sqrt{x}+2}=m$$

$$\Leftrightarrow \sqrt{x}-2=m(\sqrt{x}+2)$$

$$\Leftrightarrow \sqrt{x}-2=m\sqrt{x}+2m$$

$$\Leftrightarrow (m-1)\sqrt{x}+2m+2=0$$

$$\Leftrightarrow (m-1)\sqrt{x}=-(2m+2) \quad (*)$$

TH1: m = 1 thì phương trình trở thành: $0 = - (2m + 2) \Leftrightarrow m = -1$ (vô lý)

TH2: $m \neq 1$

$$(*) \Leftrightarrow \sqrt{x}=\frac{-(2m+2)}{m-1}$$

Vì $\sqrt{x} \geq 0$; $\sqrt{x} \neq 1$ nên $\begin{cases} \dfrac{-(2m+2)}{m-1} \geq 0 \\ \dfrac{-(2m+2)}{m-1} \neq 1 \end{cases} \Leftrightarrow \begin{cases} -1 \leq m < 1 \\ -(2m+2) \neq m-1 \end{cases}$

$$\Leftrightarrow \begin{cases} -1 \leq m < 1 \\ -3m-1 \neq 0 \end{cases} \Leftrightarrow \begin{cases} -1 \leq m < 1 \\ m \neq \dfrac{1}{3} \end{cases}$$

$$\Leftrightarrow \sqrt{x}=\frac{-(2Q+2)}{Q-1}$$

Kết hợp với điều kiện, ta được: $\begin{cases} \dfrac{-(2Q+2)}{Q-1} \geq 0 \\ \dfrac{-(2Q+2)}{Q-1} \neq 1 \end{cases} \Leftrightarrow \begin{cases} -1 \leq Q < 1 \\ Q \neq \dfrac{1}{3} \end{cases}$

l) Tìm x để Q đạt GTNN

$$Q=\frac{\sqrt{x}-2}{\sqrt{x}+2}$$

$$\Leftrightarrow \sqrt{x}-2=(\sqrt{x}+2).Q$$

$$\Leftrightarrow \sqrt{x}-2=\sqrt{x}.Q+2Q$$

$$\Leftrightarrow \sqrt{x}.(Q-1)+2Q+2=0$$

TH1: Q = 1 thì phương trình trở thành: $0 + 2.1 + 2 = 0 \Leftrightarrow 4 = 0$ (vô lý)

TH2: $Q \neq 1$ thì phương trình tương đương với:

$$\Leftrightarrow \sqrt{x}=\frac{-(2Q+2)}{Q-1}$$

Vì $\sqrt{x} \geq 0$; $\sqrt{x} \neq 1$ nên $\begin{cases} \dfrac{-(2Q+2)}{Q-1} \geq 0 \\ \dfrac{-(2Q+2)}{Q-1} \neq 1 \end{cases} \Leftrightarrow \begin{cases} -1 \leq Q < 1 \\ Q \neq \dfrac{1}{3} \end{cases}$

Khi Q = -1 thì x = 0. Vậy GTNN của Q là -1 tại x = 0.

Bài 3. Cho biểu thức : $B = \left(\dfrac{1}{\sqrt{x}+2} - \dfrac{1}{2-\sqrt{x}} + \dfrac{x}{x-4} \right) : \left(1 + \dfrac{4}{\sqrt{x}-2} \right)$

a) Rút gọn biểu thức

$B = \left(\dfrac{1}{\sqrt{x}+2} - \dfrac{1}{2-\sqrt{x}} + \dfrac{x}{x-4} \right) : \left(1 + \dfrac{4}{\sqrt{x}-2} \right)$

$(DKXD : x \geq 0; x \neq 4)$

$= \left(\dfrac{1}{\sqrt{x}+2} + \dfrac{1}{\sqrt{x}-2} + \dfrac{x}{\sqrt{x}^2-2^2} \right) : \left(1 + \dfrac{4}{\sqrt{x}-2} \right)$

$= \left(\dfrac{\sqrt{x}-2+\sqrt{x}+2+x}{(\sqrt{x}+2)(\sqrt{x}-2)} \right) : \left(\dfrac{\sqrt{x}-2+4}{\sqrt{x}-2} \right)$

$= \left(\dfrac{x+2\sqrt{x}}{(\sqrt{x}+2)(\sqrt{x}-2)} \right) : \left(\dfrac{\sqrt{x}+2}{\sqrt{x}-2} \right)$

$= \dfrac{\sqrt{x}(\sqrt{x}+2)}{(\sqrt{x}+2)(\sqrt{x}-2)} \cdot \dfrac{\sqrt{x}-2}{\sqrt{x}+2}$

$= \dfrac{\sqrt{x}}{\sqrt{x}+2}$

b) Tìm x để B = 1/5

$B = \dfrac{1}{5}$

$\Rightarrow \dfrac{\sqrt{x}}{\sqrt{x}+2} = \dfrac{1}{5}$

$\Leftrightarrow 5.\sqrt{x} = \sqrt{x}+2$

$\Leftrightarrow 4\sqrt{x} = 2$

$\Leftrightarrow \sqrt{x} = \dfrac{1}{2}$

$\Leftrightarrow x = \dfrac{1}{4} \ (tm)$

c) Tìm x để $B = \dfrac{1}{\sqrt{x}+3}$

$B = \dfrac{1}{\sqrt{x}+3}$

$\Rightarrow \dfrac{\sqrt{x}}{\sqrt{x}+2} = \dfrac{1}{\sqrt{x}+3}$

$\Leftrightarrow \dfrac{\sqrt{x}}{\sqrt{x}+2} = \dfrac{1}{\sqrt{x}+3}$

$\Leftrightarrow \sqrt{x}.(\sqrt{x}+3) = \sqrt{x}+2$

$\Leftrightarrow x+2\sqrt{x}-2 = 0$

$\Leftrightarrow x+2\sqrt{x}+1-3 = 0$

$\Leftrightarrow (\sqrt{x}+1)^2 - \sqrt{3}^2 = 0$

$\Leftrightarrow (\sqrt{x}+1-\sqrt{3})(\sqrt{x}+1+\sqrt{3}) = 0 \qquad (*)$

$Vi \ \sqrt{x} \geq 0 \ \forall x \in TXD \ nen \ \sqrt{x}+1+\sqrt{3} \geq 1+\sqrt{3} > 0 \forall x \in TXD$

$\Rightarrow (*) \Leftrightarrow \sqrt{x}+1-\sqrt{3} = 0$

$\Leftrightarrow \sqrt{x} = \sqrt{3}-1$

$\Leftrightarrow x = (\sqrt{3}-1)^2 \qquad (tm)$

d) Tìm x nguyên lớn nhất để B < ½

$$B < \frac{1}{2}$$

$$\Rightarrow \frac{\sqrt{x}}{\sqrt{x}+2} < \frac{1}{2}$$

$$\Leftrightarrow \frac{\sqrt{x}}{\sqrt{x}+2} - \frac{1}{2} < 0$$

$$\Leftrightarrow \frac{2\sqrt{x}-(\sqrt{x}+2)}{2(\sqrt{x}+2)} < 0$$

$$\Leftrightarrow \frac{\sqrt{x}-2}{2(\sqrt{x}+2)} < 0 \qquad (**)$$

$Vi \ \sqrt{x} \ge 0 \ \forall x \in TXD \ nen \ \sqrt{x}+2 \ge 2 > 0 \qquad \forall x \in TXD$

$$\Rightarrow 2(\sqrt{x}+2) > 0 \qquad \forall x \in TXD$$

$$\Rightarrow (**) \Leftrightarrow \sqrt{x}-2 < 0$$

$$\Leftrightarrow \sqrt{x} < 2 \Leftrightarrow 0 \le x < 4$$

Kết hợp với điều kiện xác định ta được: $0 \le x < 4$

e) **Tìm** $x \in Z$ **để** $3B \in Z$

$$3B = \frac{3\sqrt{x}}{\sqrt{x}+2} = \frac{3(\sqrt{x}+2)-6}{\sqrt{x}+2} = 3 - \frac{6}{\sqrt{x}+2}$$

$$3B \in Z \Leftrightarrow 3 - \frac{6}{\sqrt{x}+2} \in Z \Leftrightarrow \frac{6}{\sqrt{x}+2} \in Z$$

$$\Rightarrow 6 \vdots \sqrt{x}+2$$

$$\Rightarrow \sqrt{x}+2 \in U(6)$$

$Vi \ \sqrt{x} \ge 0 \ \forall x \in TXD \ nen \ \sqrt{x}+2 \ge 2 > 0 \qquad \forall x \in TXD$

$$\Rightarrow \sqrt{x}+2 \in \{2;3;6\}$$

$TH1: \sqrt{x}+2 = 2 \Leftrightarrow \sqrt{x} = 0 \Leftrightarrow x = 0 \ (tm)$

$TH2: \sqrt{x}+2 = 3 \Leftrightarrow \sqrt{x} = 1 \Leftrightarrow x = 1 \ (tm)$

$TH3: \sqrt{x}+2 = 6 \Leftrightarrow \sqrt{x} = 4 \Leftrightarrow x = 16 \ (tm)$

Vậy giá trị x nguyên cần tìm là $\{0;1;16\}$.

f) **Tìm** x **để** $3B \in Z$

$$3B = \frac{3\sqrt{x}}{\sqrt{x}+2} = \frac{3(\sqrt{x}+2)-6}{\sqrt{x}+2} = 3 - \frac{6}{\sqrt{x}+2}$$

$$3B \in Z \Leftrightarrow 3 - \frac{6}{\sqrt{x}+2} \in Z \Leftrightarrow \frac{6}{\sqrt{x}+2} \in Z$$

$$\Rightarrow 6 \vdots \sqrt{x}+2$$

$$\Rightarrow \sqrt{x}+2 \in U(6)$$

$Vi \sqrt{x} \geq 0 \ \forall x \in TXD \ nen \ \sqrt{x} + 2 \geq 2 > 0 \qquad \forall x \in TXD$

$\Rightarrow \sqrt{x} + 2 \in \{2; 3; 6\}$

$TH1: \sqrt{x} + 2 = 2 \Leftrightarrow \sqrt{x} = 0 \Leftrightarrow x = 0 \ (tm)$

$TH2: \sqrt{x} + 2 = 3 \Leftrightarrow \sqrt{x} = 1 \Leftrightarrow x = 1 \ (tm)$

$TH3: \sqrt{x} + 2 = 6 \Leftrightarrow \sqrt{x} = 4 \Leftrightarrow x = 16 \ (tm)$

Vậy giá trị x cần tìm là $\{0; 1; 16\}$.

g) Tìm m để có x thỏa mãn B = 1/m

$B = \dfrac{1}{m} \Leftrightarrow \dfrac{\sqrt{x}}{\sqrt{x} + 2} = \dfrac{1}{m} \qquad (m \neq 0)$

$\Leftrightarrow m.\sqrt{x} = \sqrt{x} + 2$

$\Leftrightarrow (m - 1).\sqrt{x} = 2$

Để phương trình có nghiệm thì $\begin{cases} m - 1 \neq 0 \\ \dfrac{2}{m-1} \geq 0 \\ \dfrac{2}{m-1} \neq 2 \end{cases} \Leftrightarrow \begin{cases} m \neq 1 \\ m \geq 1 \\ m - 1 \neq 1 \end{cases} \Leftrightarrow \begin{cases} m > 1 \\ m \neq 2 \end{cases}.$

h) Tìm GTNN của B

$B = \dfrac{\sqrt{x}}{\sqrt{x} + 2}$

$\Leftrightarrow B.(\sqrt{x} + 2) = \sqrt{x}$

$\Leftrightarrow (B - 1).\sqrt{x} = -2B \qquad (***)$

$TH1: B = 1 \Rightarrow (***) \Leftrightarrow 0 = -2 \ (Voly)$

$TH2: B \neq 1 \Rightarrow (***) \Leftrightarrow \sqrt{x} = \dfrac{-2B}{B-1}$

$Vi \sqrt{x} \geq 0; \sqrt{x} \neq 2 \ \forall x \in TXD$

$\Rightarrow \begin{cases} \dfrac{-2B}{B-1} \geq 0 \\ \dfrac{-2B}{B-1} \neq 2 \end{cases} \Leftrightarrow \begin{cases} 0 \leq B < 1 \\ -2B \neq 2(B-1) \end{cases} \Leftrightarrow \begin{cases} 0 \leq B < 1 \\ B \neq \dfrac{1}{2} \end{cases}$

$Khi \ B = 0 \ thi \ \sqrt{x} = 0 \Leftrightarrow x = 0 \ (tm).$

Vậy GTNN của B là 0 tại x = 0.

Bài 3. Cho biểu thức $A = \dfrac{\sqrt{x}}{\sqrt{x} - 5} - \dfrac{10\sqrt{x}}{x - 25} - \dfrac{5}{\sqrt{x} + 5}$

a) Rút gọn biểu thức A

$$A = \frac{\sqrt{x}}{\sqrt{x}-5} - \frac{10\sqrt{x}}{x-25} - \frac{5}{\sqrt{x}+5}$$

$(DKXD: x \geq 0; \ x \neq 25)$

$$= \frac{\sqrt{x}}{\sqrt{x}-5} - \frac{10\sqrt{x}}{\sqrt{x}^2-5^2} - \frac{5}{\sqrt{x}+5}$$

$$= \frac{\sqrt{x}}{\sqrt{x}-5} - \frac{10\sqrt{x}}{(\sqrt{x}-5)(\sqrt{x}+5)} - \frac{5}{\sqrt{x}+5}$$

$$= \frac{\sqrt{x}(\sqrt{x}+5) - 10\sqrt{x} - 5(\sqrt{x}-5)}{(\sqrt{x}-5)(\sqrt{x}+5)}$$

$$= \frac{x+5\sqrt{x} - 10\sqrt{x} - 5\sqrt{x} + 25}{(\sqrt{x}-5)(\sqrt{x}+5)}$$

$$= \frac{x-10\sqrt{x}+25}{(\sqrt{x}-5)(\sqrt{x}+5)}$$

$$= \frac{x - 2.\sqrt{x}.5 + 5^2}{(\sqrt{x}-5)(\sqrt{x}+5)} = \frac{(\sqrt{x}-5)^2}{(\sqrt{x}-5)(\sqrt{x}+5)}$$

$$= \frac{\sqrt{x}-5}{\sqrt{x}+5}$$

b) Tính A khi x = 9

$$A = \frac{\sqrt{9}-5}{\sqrt{9}+5} = \frac{3-5}{3+5} = \frac{-2}{8} = \frac{-1}{4}$$

c) Tìm x để A < 1/3

$$A < \frac{1}{3}$$

$$\Leftrightarrow \frac{\sqrt{x}-5}{\sqrt{x}+5} < \frac{1}{3}$$

$$\Leftrightarrow \frac{\sqrt{x}-5}{\sqrt{x}+5} - \frac{1}{3} < 0$$

$$\Leftrightarrow \frac{3(\sqrt{x}-5) - (\sqrt{x}+5)}{3(\sqrt{x}+5)} < 0$$

$$\Leftrightarrow \frac{2\sqrt{x}-20}{3(\sqrt{x}+5)} < 0$$

$$\Leftrightarrow \frac{\sqrt{x}-10}{3(\sqrt{x}+5)} < 0 \qquad (**)$$

$Vi \ \sqrt{x} \geq 0 \ \ \forall x \in TXD \ \ nen \ \ \sqrt{x}+5 \geq 5 > 0$

$\Rightarrow (**) \Leftrightarrow \sqrt{x}-10 < 0 \Leftrightarrow 0 \leq x < 100$

Kết hợp với điều kiện xác định, suy ra:

$$\begin{cases} 0 \leq x < 100 \\ x \neq 25 \end{cases}$$

d) Tìm $x \in Z$ để $A < \frac{-1}{2}$

$$A = \frac{\sqrt{x}-5}{\sqrt{x}+5}$$

$$A < \frac{-1}{2} \Rightarrow \frac{\sqrt{x}-5}{\sqrt{x}+5} < \frac{-1}{2}$$

$$\Leftrightarrow \frac{\sqrt{x}-5}{\sqrt{x}+5} + \frac{1}{2} < 0$$

$$\Leftrightarrow \frac{2(\sqrt{x}-5) + \sqrt{x}+5}{2(\sqrt{x}+5)} < 0$$

$$\Leftrightarrow \frac{3\sqrt{x}-5}{2(\sqrt{x}+5)} < 0 \qquad (*)$$

$Vi \ \sqrt{x} \geq 0 \ \ \forall x \in TXD \ \ nen \ \ \sqrt{x}+5 \geq 5 > 0 \qquad \forall x \in TXD$

$\forall x \in TXD \Rightarrow (*) \Leftrightarrow 3\sqrt{x}-5 < 0$

$$\Leftrightarrow \sqrt{x} < \frac{5}{3}$$

$$\Leftrightarrow 0 < x < \frac{25}{9}$$

Kết hợp với điều kiện xác định và x nguyên, suy ra giá trị x cần tìm là x = 1; 2.

e) So sánh A với 2

$$A - 2 = \frac{\sqrt{x}-5}{\sqrt{x}+5} - 2$$

$$= \frac{\sqrt{x}-5-2(\sqrt{x}+5)}{\sqrt{x}+5}$$

$$= \frac{-(\sqrt{x}+15)}{\sqrt{x}+5}$$

Nhận xét:

$$Vi \; \sqrt{x} \geq 0 \;\; \forall x \in TXD \; nen \begin{cases} \sqrt{x}+5 \geq 5 > 0 \\ \sqrt{x}+15 \geq 15 > 0 \end{cases}$$

$$\Rightarrow \frac{-(\sqrt{x}+15)}{\sqrt{x}+5} < 0 \qquad \forall x \in TXD$$

$$hay \quad A - 2 < 0 \qquad \forall x \in TXD$$

$$\Rightarrow \quad A < 2 \quad \forall x \in TXD$$

f) Tìm x để $A = \dfrac{2\sqrt{x}}{3}$

$$\Rightarrow \frac{\sqrt{x}-5}{\sqrt{x}+5} = \frac{2\sqrt{x}}{3}$$

$$\Leftrightarrow 3.(\sqrt{x}-5) = 2\sqrt{x}.(\sqrt{x}+5)$$

$$\Leftrightarrow 3\sqrt{x}-15 = 2\sqrt{x}.\sqrt{x}+10\sqrt{x}$$

$$\Leftrightarrow 3\sqrt{x}-15 = 2x+10\sqrt{x}$$

$$\Leftrightarrow 2x + 7\sqrt{x} + 15 = 0$$

$$\Leftrightarrow x + \frac{7}{2}\sqrt{x} + \frac{15}{2} = 0$$

$$\Leftrightarrow x + 2.\frac{7}{4}\sqrt{x} + \left(\frac{7}{4}\right)^2 + \frac{15}{2} - \frac{49}{16} = 0$$

$$\Leftrightarrow \left(\sqrt{x}+\frac{7}{4}\right)^2 - \frac{71}{16} = 0$$

$$\Leftrightarrow \left(\sqrt{x}+\frac{7}{4}-\frac{\sqrt{71}}{4}\right)\left(\sqrt{x}+\frac{7}{4}+\frac{\sqrt{71}}{4}\right) = 0 \quad (*)$$

$$Vi \; \sqrt{x} \geq 0 \;\; \forall x \in TXD \; nen \; \sqrt{x}+\frac{7}{4}+\frac{\sqrt{71}}{4} > 0.$$

$$(*) \Leftrightarrow \sqrt{x}+\frac{7}{4}-\frac{\sqrt{71}}{4} = 0$$

$$\Leftrightarrow \sqrt{x} = -\frac{7}{4}+\frac{\sqrt{71}}{4} = \frac{\sqrt{71}-7}{4}$$

$$\Leftrightarrow x = \frac{\left(\sqrt{71}-7\right)^2}{16} \qquad (tm)$$

g) Tìm $x \in Z$ **để** $\dfrac{1}{2}A \in Z$

Để $\dfrac{1}{2}A \in Z$ thì $\begin{cases} A \in Z \\ A \vdots 2 \end{cases}$.

$$A = \frac{\sqrt{x}-5}{\sqrt{x}+5} = \frac{\sqrt{x}+5-10}{\sqrt{x}+5} = 1 - \frac{10}{\sqrt{x}+5}$$

$$A \in Z \Leftrightarrow \frac{10}{\sqrt{x}+5} \in Z \Leftrightarrow 10 \vdots \sqrt{x}+5$$

$$\Rightarrow \sqrt{x}+5 \in U(10)$$

$$Vi \; \sqrt{x} \geq 0 \;\; \forall x \in TXD \;\; nen \;\; \sqrt{x}+5 \geq 5 \quad \forall x \in TXD$$

$$\Rightarrow \sqrt{x} + 5 \in \{5; 10\}$$

$$TH1 : \sqrt{x} + 5 = 5 \Leftrightarrow \sqrt{x} = 0 \Leftrightarrow x = 0 \ (tm)$$

$$TH2 : \sqrt{x} + 5 = 10 \Leftrightarrow \sqrt{x} = 5 \Leftrightarrow x = 25 \ (loai)$$

Khi x = 0 thì A = -1 không chia hết cho 2. Vì vậy, không tồn tại giá trị x thỏa mãn yêu cầu bài toán.

h) Tìm x để $A \in Z$

$$A = \frac{\sqrt{x} - 5}{\sqrt{x} + 5} = \frac{\sqrt{x} + 5 - 10}{\sqrt{x} + 5} = 1 - \frac{10}{\sqrt{x} + 5}$$

$$Vi \ \sqrt{x} \ge 0 \ \forall x \in TXD \quad nen \ \sqrt{x} + 5 \ge 5 \quad \forall x \in TXD$$

$$\Rightarrow \frac{10}{\sqrt{x} + 5} \le 2 \qquad \forall x \in TXD$$

$$\Rightarrow -\frac{10}{\sqrt{x} + 5} \ge -2 \qquad \forall x \in TXD$$

$$\Rightarrow 1 - \frac{10}{\sqrt{x} + 5} \ge -1 \qquad \forall x \in TXD$$

$$A \in Z \Leftrightarrow \frac{10}{\sqrt{x} + 5} \in Z \Leftrightarrow 10 \vdots \sqrt{x} + 5$$

$$\Rightarrow \sqrt{x} + 5 \in U(10)$$

$$Vi \ \sqrt{x} \ge 0 \ \forall x \in TXD \quad nen \ \sqrt{x} + 5 \ge 5 \quad \forall x \in TXD$$

$$\Rightarrow \sqrt{x} + 5 \in \{5; 10\}$$

$$TH1 : \sqrt{x} + 5 = 5 \Leftrightarrow \sqrt{x} = 0 \Leftrightarrow x = 0 \ (tm)$$

$$TH2 : \sqrt{x} + 5 = 10 \Leftrightarrow \sqrt{x} = 5 \Leftrightarrow x = 25 \ (loai)$$

i) Tìm GTNN của A.

$$A = \frac{\sqrt{x} - 5}{\sqrt{x} + 5} = \frac{\sqrt{x} + 5 - 10}{\sqrt{x} + 5} = 1 - \frac{10}{\sqrt{x} + 5}$$

$$Vi \ \sqrt{x} \ge 0 \ \forall x \in TXD \quad nen \ \sqrt{x} + 5 \ge 5 \quad \forall x \in TXD$$

$$\Rightarrow \frac{10}{\sqrt{x} + 5} \le 2 \qquad \forall x \in TXD$$

$$\Rightarrow -\frac{10}{\sqrt{x} + 5} \ge -2 \qquad \forall x \in TXD$$

$$\Rightarrow 1 - \frac{10}{\sqrt{x} + 5} \ge -1 \qquad \forall x \in TXD$$

Dấu "=" xảy ra khi và chỉ khi x = 0.

Vậy GTNN của A là -1 tại x = 0.